


# Advanced/Advanced Subsidiary 

Thursday 24 May 2012 - Morning Time: 1 hour 30 minutes

Materials required for examination Items included with question papers<br>Mathematical Formulae (Pink) Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

## Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature. Check that you have the correct question paper.
Answer ALL the questions.
You must write your answer to each question in the space following the question.
Values from the statistical tables should be quoted in full. When a calculator is used, the answer should be given to an appropriate degree of accuracy.

## Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. (2).
There are 8 questions in this question paper. The total mark for this paper is 75 .
There are 24 pages in this question paper. Any blank pages are indicated.

## Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

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Turn over

1. A manufacturer produces sweets of length $L \mathrm{~mm}$ where $L$ has a continuous uniform distribution with range [15, 30].
(a) Find the probability that a randomly selected sweet has a length greater than 24 mm .

These sweets are randomly packed in bags of 20 sweets.
(b) Find the probability that a randomly selected bag will contain at least 8 sweets with length greater than 24 mm .
(c) Find the probability that 2 randomly selected bags will both contain at least 8 sweets with length greater than 24 mm .
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2. A test statistic has a distribution $B(25, p)$.

Given that

$$
\mathrm{H}_{0}: p=0.5 \quad \mathrm{H}_{1}: p \neq 0.5
$$

(a) find the critical region for the test statistic such that the probability in each tail is as close as possible to $2.5 \%$.
(b) State the probability of incorrectly rejecting $\mathrm{H}_{0}$ using this critical region.
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3. (a) Write down two conditions needed to approximate the binomial distribution by the Poisson distribution.

A machine which manufactures bolts is known to produce $3 \%$ defective bolts. The machine breaks down and a new machine is installed. A random sample of 200 bolts is taken from those produced by the new machine and 12 bolts were defective.
(b) Using a suitable approximation, test at the $5 \%$ level of significance whether or not the proportion of defective bolts is higher with the new machine than with the old machine. State your hypotheses clearly.
4. The number of houses sold by an estate agent follows a Poisson distribution, with a mean of 2 per week.
(a) Find the probability that in the next 4 weeks the estate agent sells,
(i) exactly 3 houses,
(ii) more than 5 houses.

The estate agent monitors sales in periods of 4 weeks.
(b) Find the probability that in the next twelve of these 4 week periods there are exactly nine periods in which more than 5 houses are sold.

The estate agent will receive a bonus if he sells more than 25 houses in the next 10 weeks.
(c) Use a suitable approximation to estimate the probability that the estate agent receives a bonus.

## (6)

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## Question 4 continued

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5. The queueing time, $X$ minutes, of a customer at a till of a supermarket has probability density function

$$
\mathrm{f}(x)=\left\{\begin{array}{cc}
\frac{3}{32} x(k-x) & 0 \leqslant x \leqslant k \\
0 & \text { otherwise }
\end{array}\right.
$$

(a) Show that the value of $k$ is 4
(b) Write down the value of $\mathrm{E}(X)$.
(c) Calculate $\operatorname{Var}(X)$.
(d) Find the probability that a randomly chosen customer's queueing time will differ from the mean by at least half a minute.
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6. A bag contains a large number of balls.
$65 \%$ are numbered 1
$35 \%$ are numbered 2
A random sample of 3 balls is taken from the bag.
Find the sampling distribution for the range of the numbers on the 3 selected balls.
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7. The continuous random variable $X$ has probability density function $\mathrm{f}(x)$ given by

$$
\mathrm{f}(x)=\left\{\begin{array}{cc}
\frac{x^{2}}{45} & 0 \leqslant x \leqslant 3 \\
\frac{1}{5} & 3<x<4 \\
\frac{1}{3}-\frac{x}{30} & 4 \leqslant x \leqslant 10 \\
0 & \text { otherwise }
\end{array} .\right.
$$

(a) Sketch $\mathrm{f}(x)$ for $0 \leqslant x \leqslant 10$

> (4)
(b) Find the cumulative distribution function $\mathrm{F}(x)$ for all values of $x$.
(c) Find $\mathrm{P}(X \leqslant 8)$.
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## Question 7 continued

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8. In a large restaurant an average of 3 out of every 5 customers ask for water with their meal. A random sample of 10 customers is selected.
(a) Find the probability that
(i) exactly 6 ask for water with their meal,
(ii) less than 9 ask for water with their meal.

A second random sample of 50 customers is selected.
(b) Find the smallest value of $n$ such that

$$
\mathrm{P}(X<n) \geqslant 0.9
$$

where the random variable $X$ represents the number of these customers who ask for water.

Question 8 continued
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